OCR
Oxford Cambridge and RSA

An OCR endorsed teaching and learning tool

OCR A Level Computer Science

H446 – Paper 1

3

Karnaugh maps

Unit 8

Boolean Algebra

PG ONLINE

Objectives

- Understand the correspondence between a truth table and a Karnaugh map
- Understand how to fill out a Karnaugh map for a given expression
- Understand how to group items in a Karnaugh map
- Interpret the groupings in a Karnaugh map
- Simplify Boolean expressions with two, three or four variables using a Karnaugh map

Karnaugh maps

- We have seen how to complete truth tables for logic gates and logic circuits
- How do you simplify a complex Boolean expression?
- Karnaugh maps are similar to truth tables, and provide an alternative, often easier, method of simplifying expressions

Truth tables and Karnaugh maps

- Compare the truth table with the Karnaugh map:

A	B	P
0	0	a
0	1	b
1	0	c
1	1	d

A \ B	0	1
0	a	b
1	c	d

Truth tables and Karnaugh maps

- Fill in the Karnaugh map for the truth table below:

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0		
1		

Truth tables and Karnaugh maps

- Fill in the Karnaugh map for the truth table below:

A	B	P
0	0	0
0	1	1
1	0	1
1	1	0

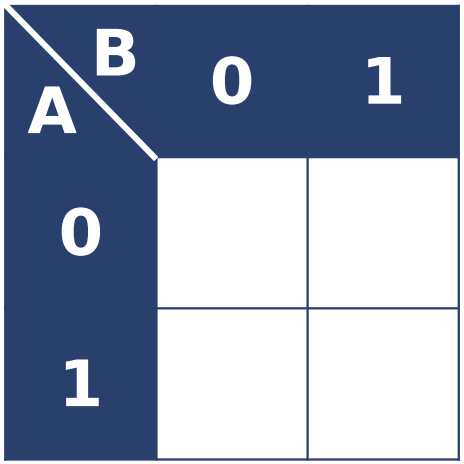
A \ B	0	1
0	0	1
1	1	0

- The top right square represents A = 0, B = 1
  - What logic gate does this represent?

Simplifying an expression

- Fill in the Karnaugh map for the expression:

$$A \vee (A \wedge B)$$



Step 1: Divide the expression into the two parts on each side of the OR symbol ($\vee$)

The whole expression is true if A is true OR if (A AND B) is true

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

A \ B	0	1
0		
1	1	1

Step 2: Write a 1 in each square for which A is true

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$

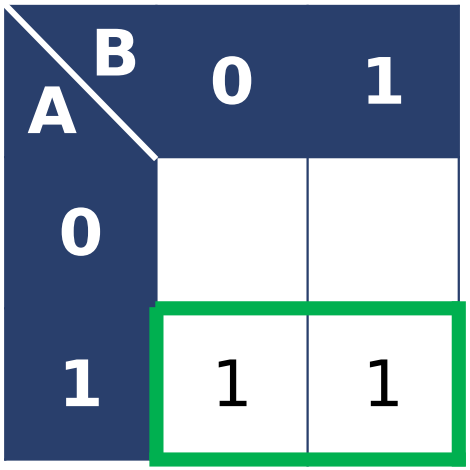
A \ B	0	1
0		
1	1	1

Step 3: Write a 1 in each square for which $A \wedge B$ is true (The square, shown in red here, is already filled in,
so nothing to do here)

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$



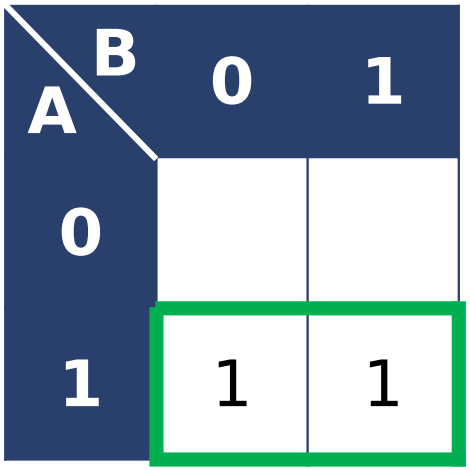
Step 4: Draw a box around the group of two adjacent 1s

What expression is represented by the group?

Simplifying an expression

- Fill in the Karnaugh map for the expression

$$A \vee (A \wedge B)$$



- The group outlined in green represents A
- This is one of the Absorption rules
- $(A \wedge B)$ has been absorbed into the A group

A 3-variable Karnaugh map

- With three variables, we combine two of them across the columns

A \ BC	00	01	11	10
0				
1				

- The terms 00, 01, 11, 10 are arranged so that in each subsequent term *only one digit changes*
  - Note carefully that they are NOT in numerical sequence

Example

- Use a Karnaugh map to represent (A ^ ¬ B) ˅ C

A \ BC	00	01	11	10
0				
1	1	1		

Step 1: Put 1 in all squares in which (A ^ ¬ B) is true

Example

- Use a Karnaugh map to represent (A ^ ¬ B) ˅ C

A \ BC	00	01	11	10
0		1	1	
1	1	1	1	

Step 2: Put 1 in all squares in which C is true

A 3-variable Karnaugh map

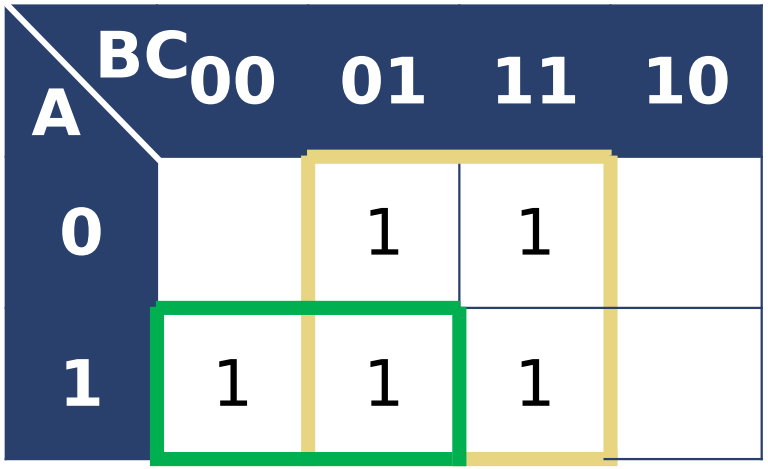
- The map represents (A ∧ ¬ B) ∨ C

A \ BC	00	01	11	10
0		1	1	
1	1	1	1	

- We now group the items in groups of 1, 2, 4 or 8 (The groups can be overlapping)
- Make each group as large as possible
- Each 1 must be included in at least one group

A 3-variable Karnaugh map

- The map represents (A ^ ¬ B) ˅ C



- The green group represents A ^ ¬ B
- The yellow group represents C
- NOTE: Never make groups of 3, 5, 6 or 7! Only groups of 1, 2, 4 or 8 are allowed

Exercise

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	00	01	11	10
0				
1				

Exercise – Step 1

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	00	01	11	10
0			1	
1				

Exercise – Step 2

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	00	01	11	10
0			1	
1		1		

Exercise – Step 3

- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	00	01	11	10
0			1	
1		1		1

Exercise – Step 4

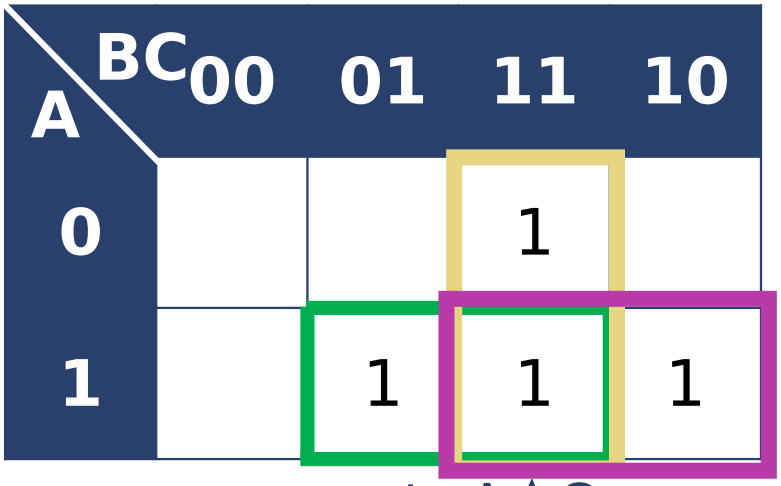
- Fill in 1s to represent:

$$(\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge B \wedge C)$$

A \ BC	00	01	11	10
0			1	
1		1	1	1

- Now group them in overlapping pairs
- Remember to make each group as large as possible
- How many groups do you end up with?

Exercise – Step 5



- The green group represents A ^ C
- The purple group represents A ^ B
- The yellow group represents B ^ C
- We have simplified the expression from:

  (¬ A ^ B ^ C) ˅ (A ^ ¬ B ^ C) ˅ (A ^ B ^ ¬ C) ˅ (A ^ B ^ C)

  to: (A ^ C) ˅ (B ^ C) ˅ (A ^ B)

Worksheet 3

- Now try the questions in **Task 1**

A 4-variable Karnaugh map

- What does this map represent?

AB \ CD	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			

A 4-variable Karnaugh map

- What does this map represent?

AB \ CD	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			

- Notice that the green group “wraps” but is still a single group

A 4-variable Karnaugh map

- This represents: **(¬A ^ B) ˅ (¬B ^ ¬C ^ ¬D)**

AB \ CD	00	01	11	10
00	1			
01	1	1	1	1
11				
10	1			

- Notice that the green group “wraps” but is still a single group

Worksheet 3

- Now try the questions in **Task 2**

Plenary

- Karnaugh maps provide a method of simplifying expressions
- This is often an easier method than using Boolean algebra (and more fun...)
- You need to know both methods

Copyright

Licence agreement